Some Developments of Chinese Mathematics in the Computer Age

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Wu's research covers many aspect of mathematics, two of which deserve special attention. In the fifties, Wu made groundbreaking contribution to topology by discovering the Wu class and Wu formulas. After 50 years, these classic results are still used, e.g. by Fields Medal recipient E. Witten (1999). Since 1975 Wu devoted himself to the creation of a new discipline which he called Mathematics Mechanization. The Chinese ancient mathematics is constructive and computational with results mainly expressed as algorithm readily adapted to modern computers. Inspired by this, Wu developed a theory of zero-set structure of polynomial systems, known as Wu's method. The method had been applied with great success to Mechanical Geometry Theorem-Proving, which is considered as a landmark in the field of Automated Reasoning. Wu also applied his method to mechanism design, robotics, CAGD, etc. Wu has received the following awards for his scientific contribution: First Prize Chinese National Natural Science Award, 1956; Tan Kah-kee (Chen Jia-geng) Prize in Math-Physical Sciences, 1993; Distinguished Scientist Award, Qui Shi Science and Technology Foundation Hongkong, 1994; Herbrand Award on Automated Deduction, 1997; National Supreme Award of Science and Technology, China, 2000.

1. Some Characteristic Features of Chinese Ancient Mathematics

The present-day mathematics is governed by the deductive axiomatic treatment with theorem-proving as its main concern which had its origin in the ancient Greek mathematics represented by the Euclid's << Elements of Geometry >> of 3cB.C.. In contrast to this the ancient Chinese Mathematics paid little attention to theorem-proving and had even no such notions of axioms, theorems, and proofs. In fact, the Chinese ancient mathematics was rather applications-oriented, with problem-solving as its main concern, and the problems to be solved arose usually from practice, the rudimentary commerce of goods exchange, the area and capacity measuration, the reconstructions, the official administrations, etc.

As the known data of the problem to be solved and the resulting values to be sought for should be connected by some kind of equations, naturally and most frequently polynomial ones, so solving of polynomial equations became the main concern of Chinese scholars for thousands of years in ancient times.

The most important Chinese classics in mathematics which had been preserved up to the present day are universally recognized to be the <<Nine Chapters of Arithmetic>> completed in 1c B.C. and its <<Annotations>> in year 263 A.D. due to Liu Hui in the Period of Three Kingdoms (220-265 A.D.). Below we shall denote these two classics by <<NC>> and <<AN>> respectively.

Already in <<NC>> there appeared methods for problems equivalent to the solving of simple linear equations and simultaneous linear systems of equations in the present day. There appeared also methods of square and cubic root-extraction equivalent to the present-day solving of simple quadratic and cubic equations. Since very remote times China had a perfect place-valued decimal system of positive integers, however large it may be. In solving the above-mentioned equations corresponding to the completion of some kinds of computations, the ancient Chinese scholars had successively enlarged the number system of positive integers to fractions, to negative numbers, and to (root-extraction)-irrational numbers. Liu Hui in << AN>> even introduced the notion of infinite place-valued decimal numbers together with some limit concept and apparatus so that a complete real-number system was already arrived in that time. We have to point out that in Europe it was only in the later half of 19th century that the real number system was completed in diverse involved ways.

From the time of <<NC>> and <<AN>> onwards the solving of polynomial equations had been incessantly developed in China and cultivated in some classic (1247) of Song Dynasty (960-1279 A.D.) to general methods of numerical solutions of polynomial equations of arbitrary degree in numerical coefficients.

In Song and Yuan Dynasty (1271-1368 A.D.) there occurred a creation of utmost importance, viz. the introduction of the notion of Heaven's Element, Earth's Element,etc., corresponding to unknown's in the present-day terminology. These notions rendered the previous intricate task of turning a problem into equations almost a triviality. Moreover, in treating the Heaven's Element, etc. as some new kind of numbers added to the real number system so that arithmetic operations may be done in the usual manner, there will naturally be introduced the notions corresponding to present-day polynomials and rational functions, as well as their algebraic manipulations with elimination procedure in particular. All these imply the essence of modern algebraic geometry and modern (polynomial) algebra in some sense. Moreover, these developments led to ideas and methods for the solving of systems of polynomial equations actually in arbitrary number of unknown variables.

The methods were clearly described in some classics (1299, 1303) due to the scholar Zhu Shijie of Yuan Dynasty. Though naturally there were many defects and incompleteness in these methods, they became the starting point of our general study in the present computer age.

It is very important to point out that all the methods of polynomial equations-solving occurred in our ancient mathematics were expressed in the form of Shu literally meaning rule or method which are actually equivalent to the present-day algorithm. Thus, the solving of simultaneous linear equations was expressed in the form of Rectangular-Array Shu and Positive-Negative Shu for transposition of terms. The solving of (simple) quadratic and cubic equations were expressed in the form of

(Square and Cubic) Root-Extraction Shus, (eventually with Cong Shu). The numerical solving of polynomial equations with numerical coefficients in arbitrary degree was expressed in the form of Positive-Negative Root-Extraction Shu. The solving of (simultaneous) high-degree polynomial equations is expressed in the form of Tian-Yuan Shu, 4-Element Shu, etc.. There were in fact many Shus of diverse character in our past-time classics.

Besides polynomial equations the present-day so-called Diophantine Equations were also studied in our ancient mathematics. Thus, already in the classic <<NC>> there was a problem solved by some Shu which gave the complete set of exact integer-values of the three sides of a right-angled triangle (called Gou-Gu Form) in our ancient times. In << AN>> Liu Hui gave even a logically rigorous proof of this Shu (of course in a style different from the usual Euclidean type). We remark in passing that general formulae about the integer-valued sides of right-angled triangles did not appear anywhere else in ancient times other than China until hundreds years later in Diophantine's work and without any indication of proof. In fact, most of historical works on mathematics gave false descriptions in this respect.

The calender-making of China since quite remote times led to the problem of solving some kind of congruence equations in the present-day terminology. Successive developments cultivated to the Da-Yian 1-Seeking Shu in the 1247- classic mentioned above. When the above result was transmitted later to Europe in 18th century it was henceforth known as Chinese Remainder Theorem which played an important role in modern mathematics.

The present-day so-called binomial coefficients together with some associated Diagram equivalent to the later known Pascal Triangle were already discovered in Song Dynasty around 10c A.D.. It was created with the purpose of high-degree root-extraction equivalent to the present-day solving of equations of the form $x^n = a > 0$, where n is an arbitrary positive integer. Also in Song Dynasty the great politician and great scholar Shen Kuo (1031-1095 A.D.) had created some theory and Shus dealing with formulas alike to the present-day combinatoric identities. Shen's creation had been much extended in Song and Yuan Dynasties with for example an identity which was re-discovered in recent years and was called accordingly Formula of Zhu Shijie and Van der Monde in the literature.

In conclusion, the ancient Chinese mathematics had the characteristic features of being applications-oriented and even highly practical, being computational, constructive, and algorithmic. In fact, most of important results in our ancient mathematics were expressed in the form of Shus, corresponding in general to the present-day algorithms, which may readily be turned into programs to be run on the computers, if one likes. As pointed out by D.E.Knuth, computer science may be considered as a science of algorithms. In this sense our ancient mathematics may be considered as a kind of computer mathematics. Its intrinsic value and possible influence in the nowadays computer age are quite clear.

The brilliant development of our ancient mathematics declined unfortunately during the Ming Dynasty (1368-1644 A.D.). It was replaced henceforth by the western mathematics of entirely different characteristics. However, the spirit of our ancient mathematics had been revived in China in recent years. In fact, in Institute of Systems Science of our Academy of Sciences, it was established in year 1990 a Research Center bearing the name of Mathematics Mechanization (abbr. MMRC). Through the efforts of members of MMRC and their collaborators spread over a vast part of China, important achievements along the line of thought and spirit of algorithmic method of our ancestors had been done as shown in the next section.

2. Mathematics-Mechanization Studies of China in the Computer Age

Being inspired by the ideas, methods, and achievements of our ancient mathematics, the present author had begun to apply computers to the study of mathematics toward the end of 70th in the last century.

Everybody had learned since his or her study in primary schools how difficult and intricate for the proving of geometry theorems. under the influence of ancient Chinese mathematics the present author tried in the end of year 1976 to find a mechanical way of proving geometry theorems. For this purpose we first turned to algebraic forms the geometry theorems by means of coordinates. After several months of painstaking trials we ultimately succeeded in discovering some algorithmic way of proving some essential part of elementary geometry theorems by algebraic manipulations. Since that time hundreds of difficult geometry theorems had been trivially proved or even discovered on the computers by this method. Later on our algorithmic method of geometry theorem-proving had evolved to a somewhat general movement of Mathematics-Mechanization with polynomial equations-solving as one of its main concerns.

Following the line of thought of our ancestors with some techniques borrowed from modern mathematics we have been able to give a general algorithmic method of solving arbitrary system of polynomial equations in the case of characteristic zero. The results are expressed in the form of various Zero-Decomposition Theorems about Zero(PS), the Zero-Set of a polynomial system PS. These theorems permit to give a complete explicit determination of the solutions or zeros of an arbitrary polynomial system of equations PS=0 in case of characteristic zero. Our method of mechanical proving of geometry theorems is actually an application of the above general method of polynomial equations-solving. As a further application, we have shown how to determine in some mechanical way the explicit forms of unknown relations known to exist .

The above method of polynomial equations-solving had been extended to the differential case. Thus, for some systems of algebraic-differential equations DPS=0 there are also Zero-Decomposition Theorems which permit to give a complete set of solutions or zeros in certain sense of such systems. As in the algebraic case, the method had been applied to the mechanical proving of differential-geometry theorems and to the automatic discovery of unknown differential relations. In particular, this had been applied to the automatic discovery of Newton's Inverse-Square Law from the Kepler's Observational Laws. Besides, the method had been applied with great success for the determination of complete set of solutions of soliton-type of a large number of partial differential equations occurring in physics and other realms of natural sciences. All these were achieved by a unique method, while in the literature such solutions can only be found for each individual partial differential equation by some intricate method peculiar only to that equation alone, even often with incomplete set of possible solutions.

A (projective) complex algebraic variety is defined as the (homogeneous) zero-set of (homogeneous) polynomial system. For projective varieties with no singularities, there may be defined the celebrated Chern Classes or Chern numbers via the associated tangent bundle of the smooth non-singular variety in question. For a variety with singularities for which tangent bundle is not defined, then the known method of extending the notions of Chern classes and Chern numbers is a very intricate one and is actually impossible to determine them explicitly in even very simple concrete cases. However, our method of polynomial equations-solving had permitted us to define such generalized Chern Classes and Chern Numbers in a quite simple and natural way which are even explicit and computational. Thus, the well-known Miyaoka-Yau Inequality between Chern Numbers which are known only for complex 2-dimensional nonsingular surfaces of certain type, had been generalized by our method through easy computations to a large number of equalities and inequalities for high dimensional hypersurfaces with arbitrary singularities. The above Miyaoka-Yau inequality is only a very particular extreme case and its truth does not require any limitation on singularities. This shows clearly the powerfulness of our general method.

Optimization problems, or min-max problems, are abound in most fields of science, engineering, and technology. Undoubtedly the solving of such problems are extremely important for our economic constructions. It is well known that such problems are usually solved by various kinds of convergent approximation methods of numerical mathematics. Such methods give usually only isolated and local optimal values. On the other hand when the objective function and the restricted conditions are all polynomial in form, which occur quite often in the nature, then our general method of polynomial equations-solving will give the global optimal values whenever they do exist. In particular, in the case of the important bilevel programming, we have shown that for certain particular examples in the literature, what the global optimal values found by some intricate numerical methods are actually not the global optimal values and are far behind the true global values determined by our method.

Our method of polynomial equations-solving in the characteristic zero case had been extended to the case of finite characteristic. This was applied to theorem-proving in finite geometries. It turns out that the results gave light to the interesting phenomena that the theorems of same hypothesis will give rise to quite different conclusions for even and odd characteristic.

In combinatorics we know that the earliest successful algorithm due to Sister Celine for the proving of combinatorial identities depends somewhat on the solving of some polynomial equations. On the other hand we have discovered some general algorithmic method of directly proving some class of combinatorial identities, including in particular the Zhu-Van der Monde identity mentioned above.

Our method of polynomial equations-solving had also been applied to the study of cryptology and some related problems.

As problems in science and technology naturally lead to polynomial equations-solving, so our general method have naturally diverse applications in science and technology. Thus, we have found new solutions of the well-known Yang-Mills Equations and Yang-Baxeter Equations in theoretical physics, not found before by other methods. Besides, we have applied our algebraic-geometry treatment to find whole set of definite-type solutions of problems in surface-fitting of CAGD. We have also dealt with various kinds of mechanisms involving four-bar linkages, manipulators, Stewart platforms, etc., eventually explicit solutions in some oncrete cases. We have also shown how to apply our method to the study of computer vision, etc., with noticeable success.

There are also further contributions in diverse directions which we are unable to enumerate here. For these we refer to the writings due to members of MMRC and their collaborators.

A general software named MMP independent of any other alike ones has also been completed with the package about our method of polynomial equations-solving as one of the central parts.

3 Future Possible Developments of Mathematics-Mechanization

The mankind is now entering a new era of information or computer age characterized in particular by the presence of powerful tool of computers. In past eras from 18th century onwards the mankind had encountered various stages of industrial revolutions which may be characterized as mechanization of physical labor. In the coming era we will be faced with a new kind of industrial revolution characterized by the mechanization of mental labor. Now mathematics is a typical mental labor. It is universally recognized as the fundamental basis of all kinds of sciences and technologies. At the same time mathematics enjoy the widest applicability to actually all kinds of activities. Hence among all kinds of mental labors, mathematics should have the highest priority and utmost urgency to be mechanized. On the other hand mathematics has the peculiarity of being clear, precise, concise, and unambiguous in its exposition which are not possessed by any other kind of mental labor. Hence among all kinds of mental labors, mathematics seems to be the easiest one to be mechanizable. Our success in the mechanization of geometry theorem-proving shows that this is really the case.

Our Mathematics Mechanization is so proposed to meet the necessity of mechanization of mental labors in the present computer age. However, our development of Mathematics-Mechanization is yet in quite nascent stage. Thus, so far theorem-proving is concerned, what we have done successfully is restricted only to the very narrow and not so important domain of elementary geometry or (local) differential geometry. However, each domain of mathematics has its own problems of theorem-proving, to be solved in some way peculiar to that domain, not necessarily reducible to solving of polynomial equations. Furthermore, it is known from mathematical logic that if the domain of mathematics in consideration is too large in some sense, then the proving of theorems in that domain may be undecidable in logician's terminology, or non-mechanizable in our terminology. On the other hand if the domain in question is too small in some sense, then the theorems to be proved may be devoid of any mathematical interest, though the domain as a whole is mechanizable. In view of this we have launched the following program to be studied in years to come:

> *Cover as much as possible the whole of mathematics by domains each of which is sufficiently small to be mechanizable, at the same time also sufficiently large to contain lot of theorems or problems of high mathematical interest.*

So far polynomial equations-solving is concerned, we remark that our method is symbolic in character, quite different from the usual numerical methods. Such symbolic methods lead usually to polynomials with milliards of terms to be out of control of the computers. It seems that to render our method practically workable, the only way is to develop some hybrid method of combining both the superiority of the symbolic method and that of the numerical method, which should of course have a solid mathematical basis.

The present-day mathematics is somewhat governed by such subjects owing much to the development of calculus which is of somewhat infinite character. On the other hand computers can deal only with objects of finite type, and work only in a finite way. Hence combinatorics as a certain kind of mathematics of finite character seems to become more and more important in the present computer age, the more so in view of information safety closely connected with national defense. Therefore we have to pay more attention to the algorithmic study of combinatorics and its alike than any time before.

We have shown how to automatically derive the Newton's Square-Reciprocity Law from the Kepler's Observational Laws. It gives a concrete example of a general method in automatic discovering of theories in mathematical form from observational or experimental facts and should be tried in diverse sciences in years to come.

Furthermore, though our general method of polynomial equations-solving have wide applications in science and technology, it waits still to become real productive forces in our technology and industry.

Finally, our software MMP needs to be improved and extended to raise up its efficiency and widen its applicability. In short, it waits to become a universal powerful tool to be used by the whole world.

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